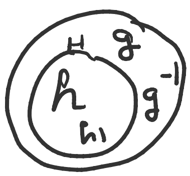
13 April 2021 10:00

Normal Subgroup

Let G be a group and H is the subgroup of G. H is said to be normal subgroup$_G$ iff $\underline{ghg^{-1}} \in H \quad \forall g \in G, \ h \in H$



$\underbrace{g * h * g^{-1}}_{h_1} \in H$

$\# \times \quad hgh^{-1} \in H \quad \forall g \in G, \ h \in H$

$\# \quad g^{-1}hg \in H \quad \forall g \in G \quad h \in H$

$\# \times \quad ghg^{-1} \in G \quad \forall g \in G \quad h \in H$

$\# \quad ghg^{-1} \in H \quad \forall g \in G \quad h \in H$

Result: $Hg = gH$

Right coset and left coset taken along same element $g \in G$ are equal only if H is a normal subgroup

Result: Let H & K be two normal subgroups of G

then $H \cap K$ is also a normal subgroup of G.

Result: If H is a normal subgroup and K is a subgroup of G then HK is also a subgroup of G.

Result: if H is a normal subgroup of G then a Quotient group is defined as $\frac{G}{H}$

Ex: $G = \{a, b, c, d, e \ldots z\}$

$H = \{b, c, d, e\}$

$Ha, Hb, Hc \ldots$

$H*a, H*b, H*c \ldots$

$\frac{G}{H}$ → normal

$Hg = gH$

$\frac{G}{H} = \{Ha, Hb, Hc \ldots Hz\}$

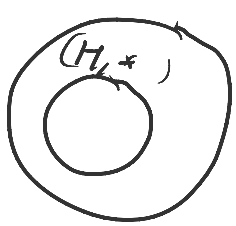
$= \{Ha, H, H, H, H, Hf, Hg \ldots Hz\}$

$= \{H, Ha, Hf \ldots Hz\}$

$= \{He, Ha, Hf \ldots Hz\}$

$= \{eH, aH, fH \ldots zH\}$

$(G, *)$



$\frac{G}{H}$ $*$

$* = +$

$1 \ldots 10$

$H\{1, 3, 5$

$\frac{G}{H} = \{H*1, H*2, H*3 \ldots\}$

$= \{H*1, H \ldots$

$H\{1, 3, 5$ $\quad \frac{}{H} = \{ \ldots \}$

$$= \{H+1, H+2, H+3 \ldots\}$$

$$(H*1)*(H*2) = H_{(1*2)}$$

$$= H_{1+2}$$

$$= H_3$$

$(Ha)(Hb) = Hab$ $\quad aHbH = abH$

aH, Ha

Result 1.

$$O\left(\frac{G}{H}\right) = \frac{O(G)}{O(H)} \qquad [G : H]$$

Result 2:- Every Quotient group of a cyclic group is cyclic

i-e if G is cyclic

then $\frac{G}{H}$ is also cyclic

then $\frac{G}{H}$ is abelian

Result 3 Quotient group is also called factor group.

Homomorphism

$f : G \to G'$ is homomorphism

[Note:- Both groups may have same or different binary composition]

if $f(ab) = f(a) \cdot f(b)$

or $f(a * b) = f(a) *' f(b)$

where '$*$' is binary composition of G and $*'$ is b-c of G'

Note :- if $f: G \to G'$

is homomorphism, one-one, onto

then f becomes isomorphism.

$G \cong G'$ → Both groups are isomorphic to each other.

{Note : $f: G \to G$ is isomorphism then it is called automorphism}

kerf : If f is homomorphism

then $\ker f \subseteq G$

s.t $f(a) = e'$ where e' is the identity element of G'

and $a \in \ker f$

$\ker f$:- $\{x ; x \in G, f(x) = e'\}$

$\ker f \neq \phi$ $\because$ $f(e) = e'$

$e \in \ker f$

optional

fundamental theorem of Homomorphism

N is a normal Subgroup of G

$\frac{G}{N}$ is a homomorphic image of

G : $f: G \to \frac{G}{N}$ is homomorphism

Note $\boxed{G \cong \ker f}$